**MFT**

\[ h_i = \sum_j J_{ij} S_i \langle B_j(\beta S_i h_j) \rangle h_j \]

Self-consistent MFT Eqs.

*Good if \( \sum_j J_{ij} S_j = \bar{H}_i \approx \bar{h}_i \) doesn't fluctuate much

- Better if each spin interacts w/ many others
  
  (law of large numbers\footnote{Note: Large \# spins} = correlated lattice)

  e.g. \( d = 3 \) better than \( d = 2 \)

For FM \( h_i = h S_i \) on periodic lattice w/ fixed coord. \#.

\[ h = 2JS B_3(\beta S h) \quad (t = \text{# n.n.'s}) \]

write \( x = \beta S h \)

\[ x = \phi \left( \frac{2JS}{k_B T} \right) B_3(x) \quad \text{Transc. eq.} \]

Non-zero solution

\[ h < T \]

appears if \( \frac{2JS^2}{k_B T} B_3(h) > 1 \)

\[ k_B T_c = \frac{2JS(5s+1)}{3} \]
How does \( M(T) \) curve? \[
M = g \mu_B \langle S^z \rangle = g \mu_B \frac{h}{2J} x h
\]
\[
h = 2J \langle S^z \rangle
\]

effective exchange field \( x \sim M \sim \exp (-\frac{x}{T_c}) \)

Interesting: \( M(T) \) cannot be analytic (No analytic fit)

\( x = 0 \) \( T \rightarrow T_c \)

\( x \rightarrow \infty \) \( T \rightarrow T_c \)

\[
x \approx \frac{\beta J S^2}{k_B T} B_3(x)
\]

\[
x \rightarrow 0 \quad h \rightarrow T \sim T_c
\]

\[
x = \frac{\beta J S^2}{k_B T} \left[ B_3'(x) + \frac{1}{6} B_3''(x) x^3 \right]
\]

\[
1 \quad T = T_c
\]

so

\[
x \approx \frac{T_c}{T} x \sim -\frac{\beta J S^2 |B_3'(x)| x^3}{k_B T_c}
\]

\[
x \approx (T_c - T)^{\frac{1}{2}} \frac{T_c - T}{T_c}
\]

\[
M \sim h \sim x \sim \text{const} \sqrt{1 - \frac{T}{T_c}}
\]
This was slope $\frac{\partial M}{\partial T}$ diverge at $T\rightarrow T_c$!

Susceptibility $\rightarrow$ next page
\[ \chi = \left. \frac{\partial M}{\partial H} \right|_{H=0} \quad h \rightarrow h + g/0 H \]

\[ T > T_c \quad \text{expect} \quad \chi < \infty. \quad (\chi = \infty \text{ at } T = T_c \text{ sing.}) \]

\[ h = z J S B_0 (\beta S (h + g/0 H)) \]

\[ \approx z J S B'_0 (\beta S h + \beta S g/0 H) \]

\[ h (1 - \frac{T_c}{T}) \approx \beta z J S^2 B'_0 (0) g/0 H \]

\[ \approx \left( \frac{\beta S^2}{g'0} \right) \left( \frac{S (S+1)}{2z} \right) g/0 H \approx g/0 H \]

\[ h \approx g/0 H \left( \frac{T_c}{T - T_c} \right) \]

\[ \Rightarrow M = \frac{g/0 h}{z J} \approx \frac{g/0 H}{z J} \frac{T_c}{T - T_c} = \left( \frac{g/0}{z J} \right)^2 \frac{S (S+1)}{3 k_B (T - T_c)} H \]

\[ i.e. \quad \chi = \frac{\chi_0}{1 - \frac{T_c}{T}} = \frac{(g/0)^2 S (S+1)}{3 k_B (T - T_c)} \quad \text{diverges at } T = T_c \]

\[ \frac{\partial \chi}{\partial H_{\text{stag.}}} = \frac{\partial N}{\partial H_{\text{stag.}}} \]

\[ \text{while } \chi_{\text{rot.}} \propto \frac{\chi_0}{T + T_c} \]

For an AF, MFT shows similar divergence of \( \chi_{\text{stag.}} \).