Exotic Order and Criticality in Quantum Matter

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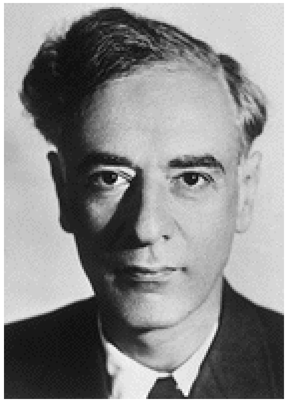
Principles of Condensed Matter Physics – thanks to Landau



- Symmetry classification of phases
  - -different phases of matter are distinguished by their degree of broken symmetry (order parameters)
  - - phase transitions are described by fluctuations and condensation of the order parameter (LGW expansion)
- Fermi liquid theory/Quasiparticle picture
  - - solids can be understood by adiabatic continuity (perturbation theory) from non-interacting band electrons – possibly in a lower-symmetry potential described by non-zero Landau order parameters

Is it really all so black and white?

Exotica

• Recent theoretical work suggests a *vast* set of “colorful” phenomena beyond the Landau paradigms

• Phases with “quantum order” definitely exist (in models)
  - No symmetry breaking: “true Mott insulators”
  - Not describable by band theory
  - Quasiparticles $\neq$ electrons, and carry emergent and/or fractional quantum numbers

• Quantum phase transitions can violate Landau paradigms
  - No Landau-Ginzburg-Wilson expansion
  - Violate “rules” for continuous transitions
  - Can have emergent and/or fractional quantum numbers

Why Care?

- Exotica is *different*
  - Heretofore unknown and subtle mechanisms of organization of quantum matter
- Is it real?
  - If it occurs in simple models (it does), Nature will have discovered and used it long before we did!
  - Strong connections to Mott insulators/transitions
    - ■ Mott behavior – localization of charge by strong interaction – does not naturally fit Landau paradigms
    - ■ Exotica appears to provide the most natural theoretical formulation of pure Mott physics
- What is the payoff?
  - -Most vexing experimental puzzles in condensed matter physics occur in proximity to Mott transitions
    - - e.g. Most experimental QCPs do not seem to fit LGW theory

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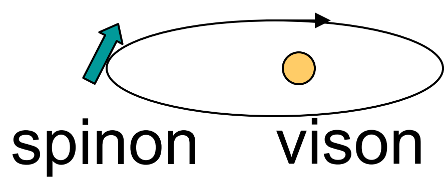
Exotic Phases

- Phenomenology: universal low-energy description of various categories of "spin liquids" in 2d and 3d are understood

  - topological phases: fully gapped 2d spin liquids with various "particles" with non-trivial braiding statistics

    e.g. Z_2:



$\psi \rightarrow -\psi$

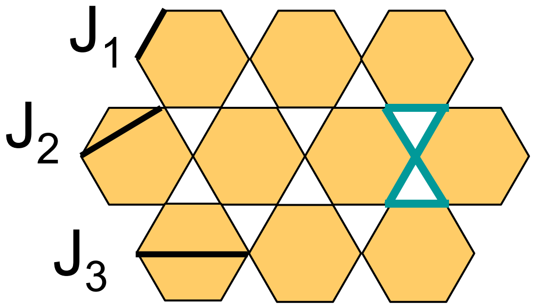
  - other "quantum ordered" phases support gapless modes, power-law correlations (U(1) spin liquid, algebraic spin liquid)

- Not just phenomenology

  - a number of lattice models can be shown to exhibit such phases, and are stable to small perturbations

Example: Generalized Kagome Ising Antiferromagnet

L.B., M.P.A. Fisher, S. Girvin, 2002
D.-N.Sheng, LB, 2004



$J_1=J_2=J_3$

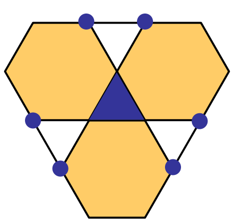
$$H_s = \frac{1}{2}\sum_{ij} J^z_{ij} S^z_i S^z_j + J^{xy}_{ij}(S^x_i S^x_j + S^y_i S^y_j)$$

$J^z \gg J^{xy}$

added by hand

$$H_{eff} = J^z \sum_{\hexagon} (S^z_{\hexagon})^2 - K \sum_{\bowtie} (S_1^+ S_2^- S_3^+ S_4^- + \text{h.c.}) + v \sum_{\bowtie} \hat{P}_{1234}$$

behaves as "spinon" with $S^z=1/2$

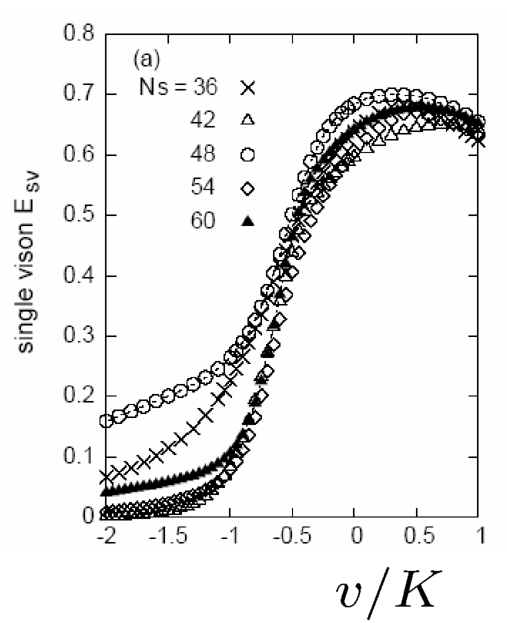


$\prod_{\bullet} \sigma^x_i = -1$ behaves as Z_2 vortex ("vison")

- If vison is gapped, spin liquid is stable

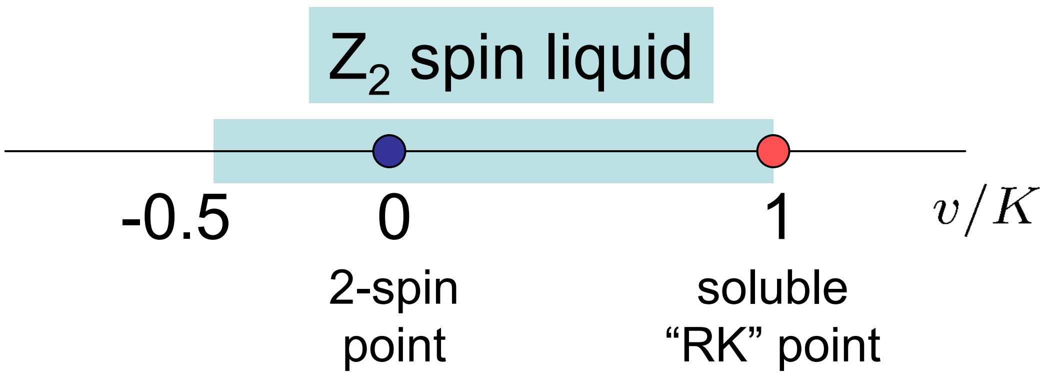
Kagome Phase Diagram

- Exact diagonalization



D.-N. Sheng, LB, 2004 (PRL in press)

APS talk: U39.00001



- Spin liquid state is stable in the two-spin limit!

  - c.f. Next talk: this model is equivalent to a **3**-dimer model on the triangular lattice (Moessner-Sondhi). Appears to have much more stable spin liquid phase than **1**-dimer model.

Other models of exotic phases

(a partial list)

- Quantum dimer models — Moessner, Sondhi; Misguich *et al*
- Rotor boson models — Motrunich, Senthil
- Pyrochlore antiferromagnet — Hermele, M.P.A. Fisher, LB
  - related model of charge frustration — D.Bergman, G. Fiete, LB, APS talk J25.00011
- Quantum loop models — Freedman, Nayak, Shtengel
- Honeycomb "Kitaev" model — Kitaev

other "sightings"

- Triangular 2+4-spin exchange model (Z_2?) — Misguich *et al*
- Kagome Heisenberg antiferromagnet (strange) — Misguich *et al*
- SU(4) Hubbard-Heisenberg model (algebraic SL?) — Assaad

...

■ Models are not crazy but contrived. It remains a huge challenge to find these phases in the lab – and develop theoretical techniques to look for them in realistic models.

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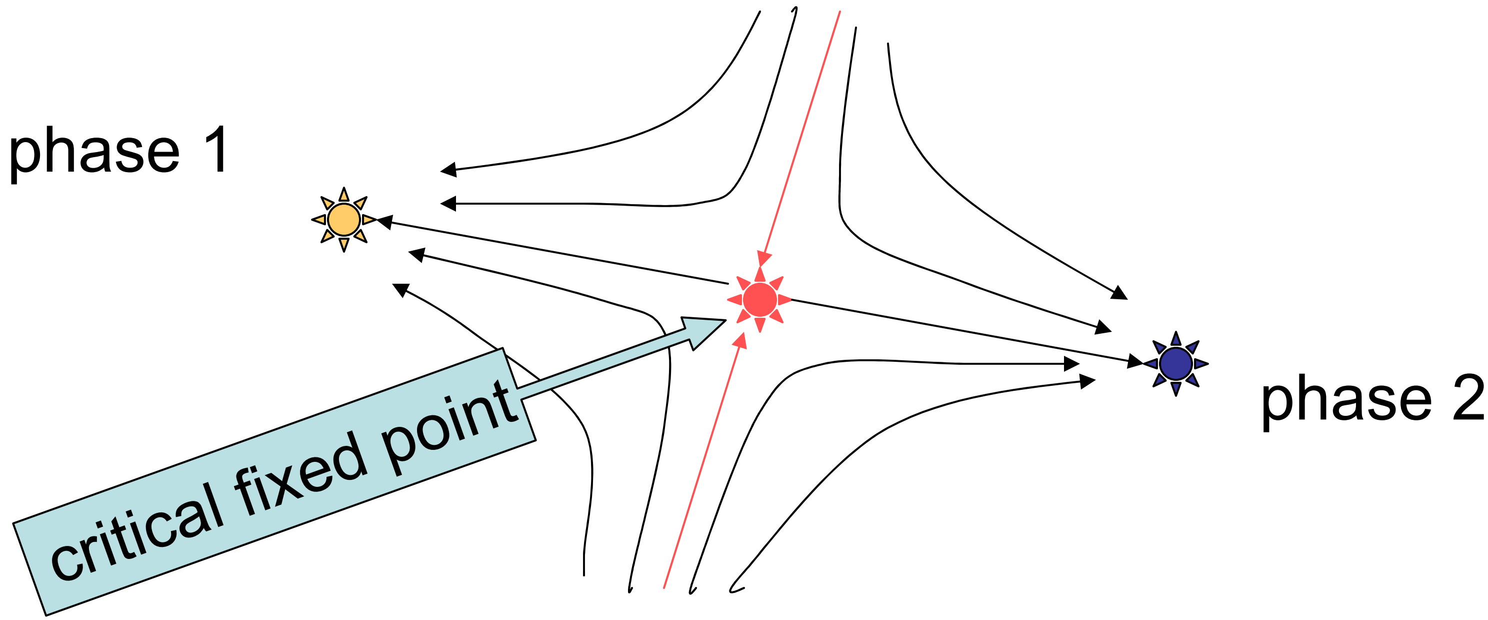
- **This occurs even when two phases are *conventional***

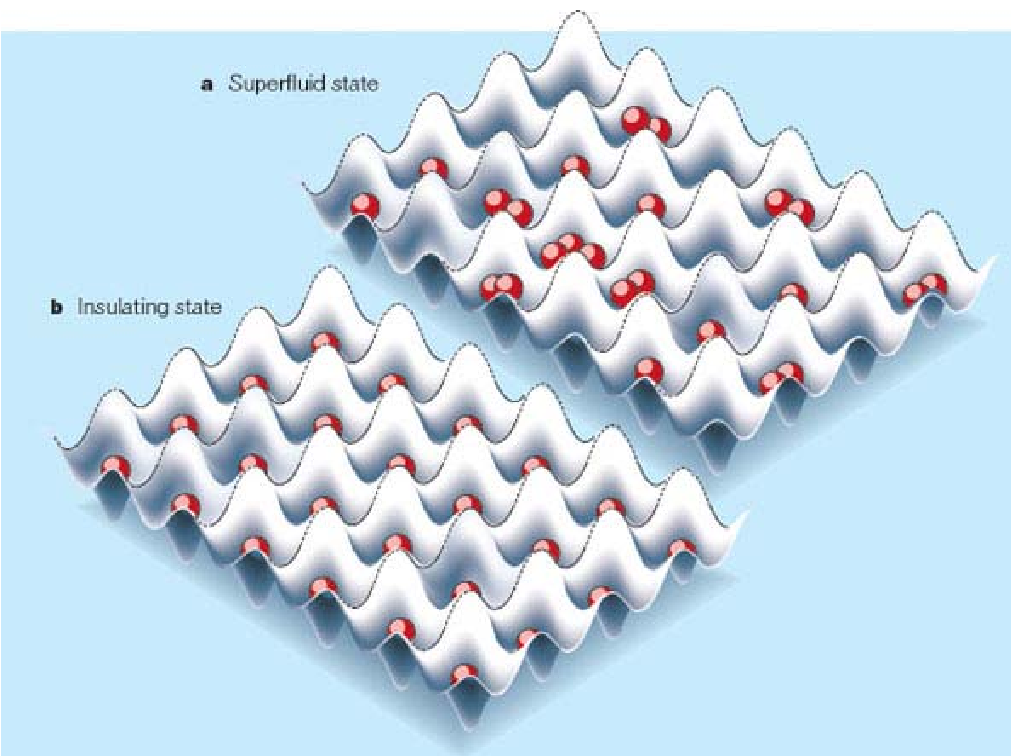
RG Theory of Continuous Critical Phenomena



- In general, expect the symmetry group of either phase is a subgroup of that at the critical fixed point.
  - LGW theory: special case where those groups coincide for the "disordered" phase.

Bose Mott Transitions

- Specialize to: Superfluid-Insulator QCPs of bosons on 2d lattices – **a vortex view**



Filling f=1: Unique Mott state w/o order

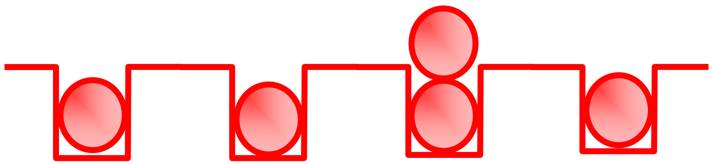
$f \neq 1$: localized bosons must order

Key problem of LGW theory: no “disordered” Mott state

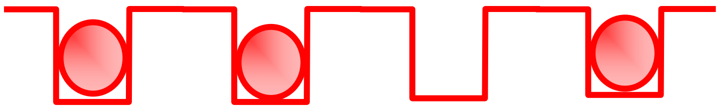
M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

Conventional Approach: from the Insulator (f=1)

Excitations:



Particles ~ $\psi^\dagger$



Holes ~ ψ

Density of particles = density of holes ⇒
"relativistic" field theory for ψ:

$$\mathcal{S} = \int d^2 r d\tau \left[|\partial_\tau \psi|^2 + |\nabla_r \psi|^2 + s|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

$$\text{Insulator} \Leftrightarrow \langle \psi \rangle = 0$$

$$\text{Superfluid} \Leftrightarrow \langle \psi \rangle \neq 0$$

- The particle/hole theory *is* LGW theory!
  - But this is possible only for f=1

Approach from the Superfluid

- Focus on *vortex* excitations

- Time-reversal exchanges vortices+antivortices
  - Expect *relativistic field theory for* φ

- Worry: vortex is a non-local object, carrying superflow
  - Resolution: *duality*

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett*. **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett*. **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev*. B **39**, 2756 (1989);

- All non-locality is accounted for by dual U(1) gauge force

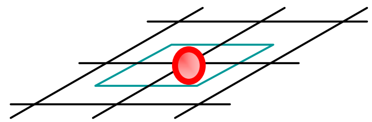
Duality

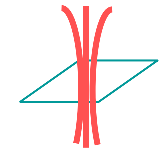
C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett*. **47**, 1556 (1981); D.R. Nelson, *Phys. Rev. Lett*. **60**, 1973 (1988); M.P.A. Fisher and D.-H. Lee, *Phys. Rev*. B **39**, 2756 (1989);

- Exact mapping from boson to vortex variables.

- Dual magnetic field B = 2πn

$$n = \frac{1}{2\pi}\vec{\nabla} \times \vec{A}$$

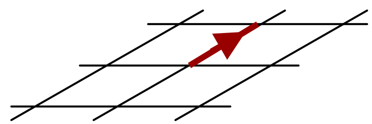




$$n = 1$$

$$\int d^2x\, B = 2\pi$$

$$\vec{\nabla}\phi = 2\pi \hat{z} \times \vec{E},$$

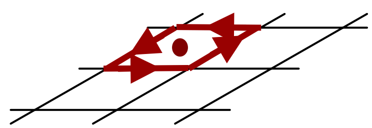


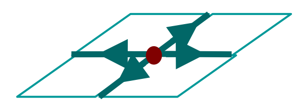
$$v_{sf} \propto \vec{\nabla}\phi$$

$$\vec{E}$$

- Vortex carries dual U(1) gauge charge

$$\vec{\nabla} \cdot \vec{E} = N$$



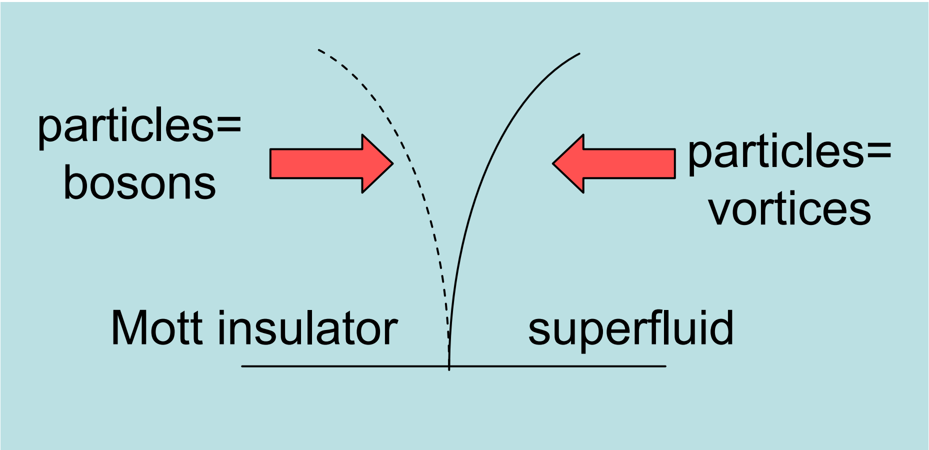


$$\oint \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi$$

$$N = 1$$

- All non-locality is accounted for by dual U(1) gauge force

Dual Theory of QCP for f=1



- Two completely equivalent descriptions
  - really one critical theory (fixed point) with 2 descriptions

C. Dasgupta and B.I. Halperin, *Phys. Rev. Lett.* **47**, 1556 (1981);

$$\mathcal{S} = \int d^3x \left[|\partial_\mu \psi|^2 + s|\psi|^2 + u|\psi|^4 \right]$$

$$\tilde{\mathcal{S}} = \int d^3x [|(\partial_\mu - iA_\mu)\varphi|^2 + \tilde{s}|\varphi|^2 + u|\varphi|^4 + \frac{1}{2e^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2]$$

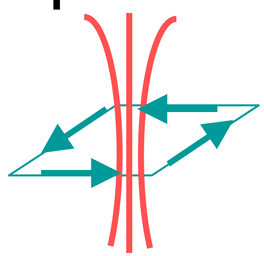
- N.B.: vortex field φ is not gauge invariant
  - not an order parameter in Landau sense

- Real significance: "Higgs" mass $|\langle\varphi\rangle|^2 A^2$ indicates Mott charge gap

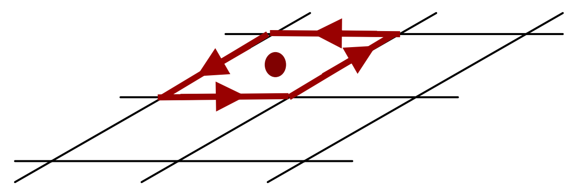
Non-integer filling $f \neq 1$

- Vortex approach now superior to Landau one
  -need not postulate unphysical disordered phase

- Vortices experience average dual magnetic field
  - physics: phase winding



Aharonov-Bohm phase

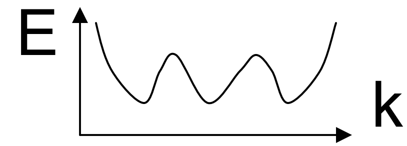


2π vortex winding

- Like type-II superconductor, vortex condensate in dual "field" must break spatial symmetries
  - vortex theory "knows" about order in the Mott state!

Vortex PSG and Order

- Vortices carry “flavor” quantum number (Hofstadter band index) and transform amongst one another under spatial symmetries: “PSG”



- Gauge-invariant bilinears transform as density wave order parameters, e.g.

$$\rho_{mn} = e^{i\pi mnf} \sum_{\ell=1}^{q} \varphi_\ell^* \varphi_{\ell+n} e^{2\pi i \ell m f} \qquad Q_{mn} = \frac{2\pi p}{q}(m,n)$$

- Vortex condensate = Mott state *always* has order
  - The order is a *secondary consequence* of Mott transition

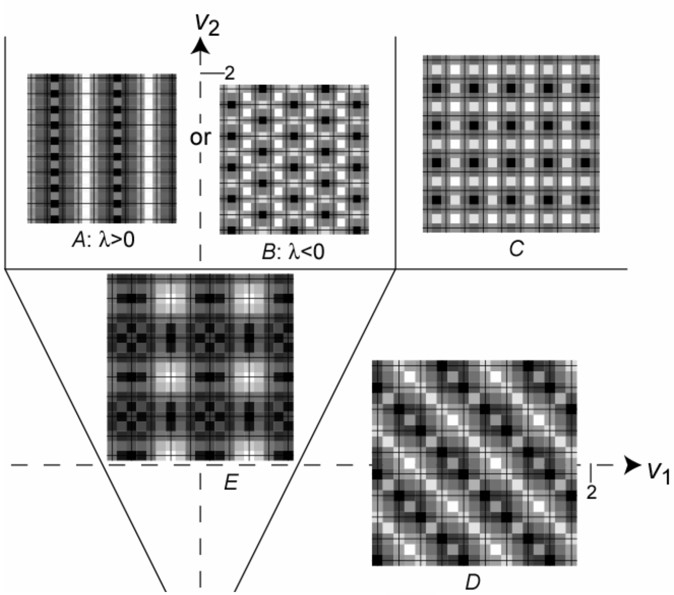
$$\rho_{mn} \propto \left| \langle \varphi \rangle \right|^2$$

Critical Theory

$$\tilde{S} = \int d^3x \Big[\frac{1}{2e^2} (\epsilon_{\mu\nu\lambda} \partial_\nu A_\lambda)^2 + \sum_\ell |(\partial_\mu - i A_\mu)\varphi_\ell|^2 + \tilde{s}|\varphi_\ell|^2 + \sum_{\ell mnp} u_{\ell mnp} \varphi^*_\ell \varphi^*_m \varphi_n \varphi_p + \cdots \Big]$$

- u_{lmnp} and H.O.T.s constrained by PSG

- "Unified" competing orders determined by simple MFT
  - -always integer number of bosons per enlarged unit cell

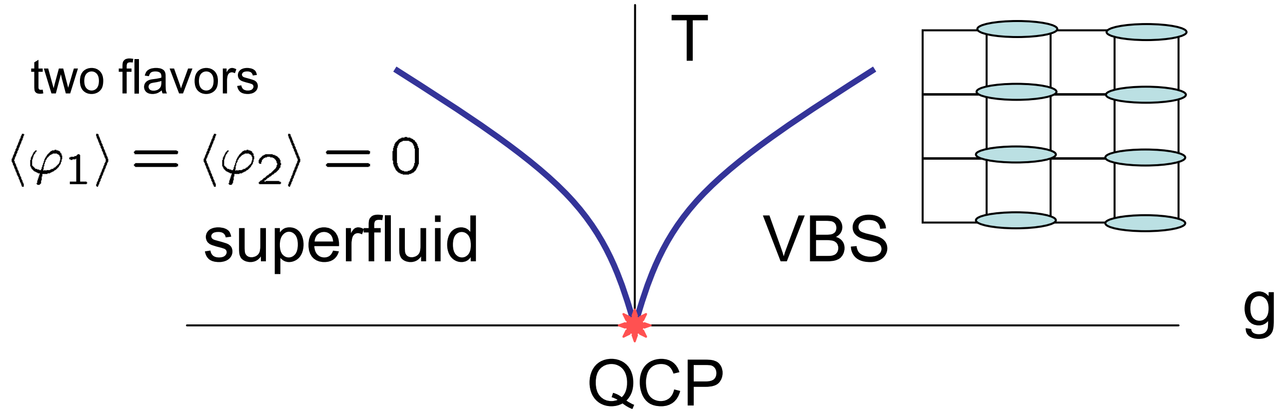


- Caveat: fluctuation effects mostly unknown

f=1/4,3/4

see T. Senthil *et al*, *Science* **303**, 1490 (2004).

Example 1: f=1/2 square lattice



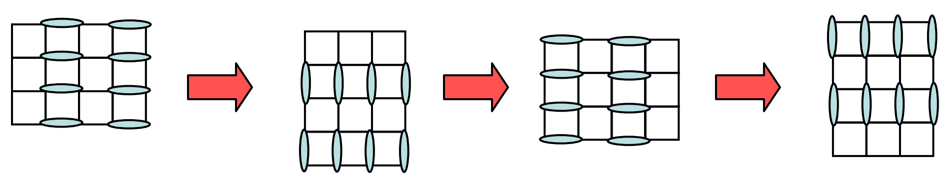
VBS order parameter

$$\Psi = \varphi_1^* \varphi_2$$

• **Not** a Landau transition: neither phase's symmetry group is a subgroup of the other

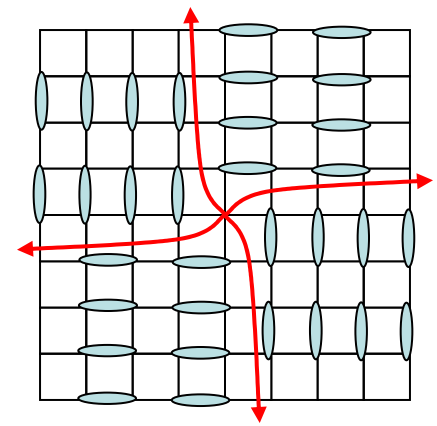
• Remarkable property of QCP: enhanced U(1) symmetry

$$\Psi \to \Psi e^{i\chi}$$



- discrete rotation amongst 4 states becomes approximately continuous at the critical point

More on f=1/2



Levin, Senthil

- Continuous rotation allows Ψ "vortex"
  - -Non-trivial "½-boson" particle!
  - -In fact, the theory can be wholly reformulated in terms of these fractional particles instead of vortices!

see T. Senthil *et al*, *Science* **303**, 1490 (2004).

"Deconfined" QCP

- some others DQCPs do not have a vortex interpretation

- Caveat:
  - -necessary VBS state does not occur in simple models
  - -so far, no numerical verification of this theory

▪ c.f. APS talks proving VBS order in Heisenberg models: S9.00001, X43.00015

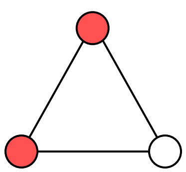
w/ O. Starykh, A. Furusaki

Example 2: f=1/2 triangular lattice

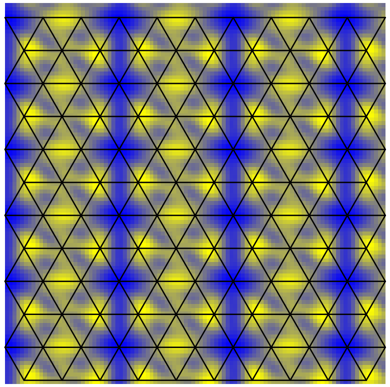
APS talk: S9.00006

A.A. Burkov and L.B., *in preparation*
A. Paramekanti, A. Vishwanath, *private comm.*

- Vortices carry 4 flavors $\varphi_{1\uparrow}, \varphi_{1\downarrow}, \varphi_{2\uparrow}, \varphi_{2\downarrow}$
- Vortex action has unexpected SU(2)×U(1) symmetry
  - much *larger* symmetry reflects *frustration: near degeneracy amongst many charge-ordered Mott states*



one of a set of 36 degenerate Mott states



- Fractionalization
  - -Again: ½-boson "vortex" excitations
  - -Also: 1-boson "skyrmion" with non-trivial charge-order "halo"
- May be possible to observe this in simple XXZ model appropriate to spinor bosons in an optical lattice

Application: Superconducting Vortex

L. Bartosch, L.B. and S. Sachdev, cond-mat/0502002

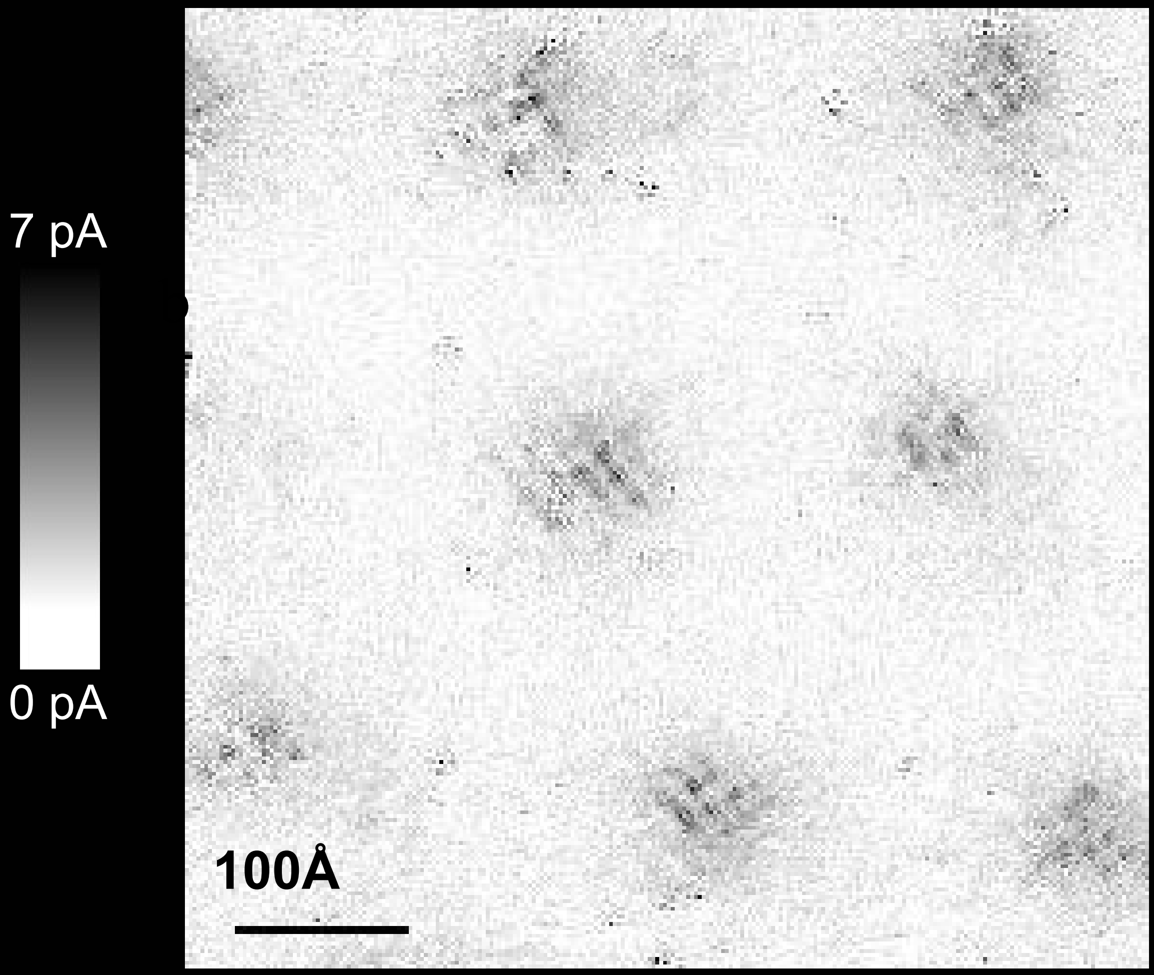
- In low-field limit, can study quantum mechanics of a single vortex localized in lattice or by disorder
  - Pinning potential selects some preferred superposition of q vortex states

$\Rightarrow$ $\rho_{mn} \neq 0$ locally near vortex

Each pinned vortex in a superconductor has a halo of density wave order over a length scale ≈ the zero-point quantum motion of the vortex. This scale diverges upon approaching the Mott insulator

- Note: we are assuming gapless quasiparticles do not drastically alter this picture

Vortex-induced LDOS of $Bi_2Sr_2CaCu_2O_{8+\delta}$ integrated from 1meV to 12meV at 4K



J. Hoffman E. W. Hudson, K. M. Lang, V. Madhavan, S. H. Pan, H. Eisaki, S. Uchida, and J. C. Davis, *Science* 295, 466 (2002).

Vortices have halos with LDOS modulations at a period ≈ 4 lattice spacings

- comparison to model of vortex dynamics gives vortex mass

$m_v < 8\ m_e$

Conclusions

- Exotic phases exist in models and are locally stable
  - Challenge is to find them in the lab and develop techniques to locate them in realistic models
- Quantum critical points exist that violate Landau's paradigm (vortex formulation is one fruitful approach)
  - These still need to be found in models
  - And better, to find the implications for experimental QCPs
- Many fundamental theoretical questions also remain, especially in fermionic and frustrated systems

Thanks: NSF The David and Lucile Packard Foundation