UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Physics

Prof. S.B. Giddings **Physics 229A** Winter 2007

Gauge Theories

ASSIGNMENT #3

Due Thursday, February 1, 2007

1. $SU(2) \times U(1)$ Chiral Lagrangians
   a) Derive the gauge boson mass from $\mathcal{L}_\Sigma$ in unitary gauge.
   b) Derive the deviation in ρ from 1 in terms of γ_1.
   c) Check the invariance of $\mathcal{L}_\Sigma$ under custodial $SU(2)$. Show that the term proportional to γ_1 is not invariant.

2. a) Assuming a lagrangian $\mathcal{L} = \mathcal{L}_{\rm gauge} - M_w^2 W^+ W^- - \frac{Mz^2}{2} Z^2$, compute the high-energy behavior of the amplitude $M(W_L W_L \to Z_L Z_L)$ for scattering of longitudinal states.

   b) Now consider the standard model lagrangian, with a minimal Higgs. Compute the extra contribution to $M(W_L W_L \to Z_L Z_L)$ from Higgs exchange and show that it improves the high energy behavior. (Hint: you may wish to use unitary gauge.)

Extra Credit — Anomalies in the standard model

As you know from field theory, symmetries can be anomalous, and an anomaly in a gauge symmetry in particular spoils the symmetry structure of the theory. Such anomalies, which in four dimensions arise from triangle diagrams, are proportional to the quantity $Tr\Big[\{T^a(R), T^b(R)\}T^c(R)\Big]$ where $T^a(R)$ is the symmetry generator in representation R, and by convention the trace is over the left-handed fermions. (So right-handed fermions contribute through the complex conjugate representation.)

a) Show that all anomalies cancel for standard model gauge symmetries.

b) Compute the anomalies in the baryon and lepton number symmetries, B and L. Find the combination of these which is non-anomalous.